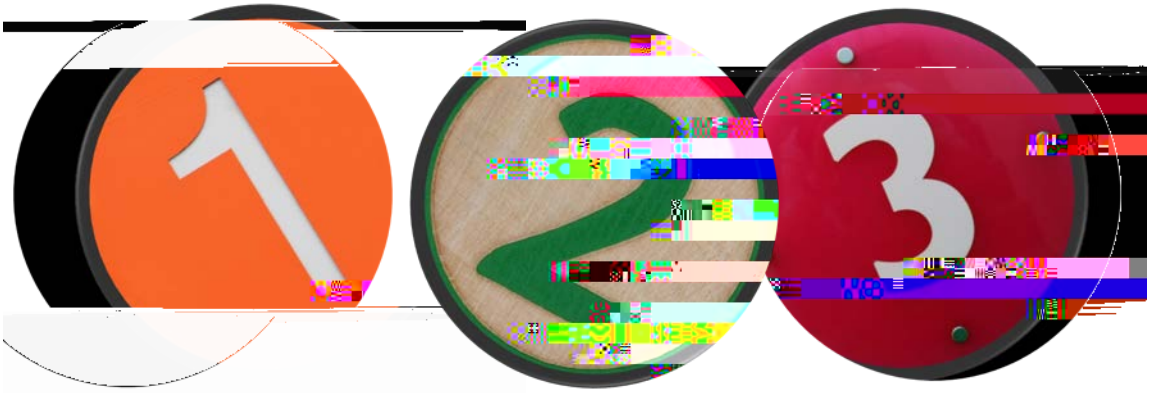
Maths Refresher

Workbook 1

This booklet will help to refresh your understanding of:

- number
- decimals, fractions and percentages
- exponents and roots



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Booklet1

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12 (continued). Activity: What do I need to get o (64h.3 (u)5.6 (o)- 0 Td (12mn)13.1 (....84 Tw5...84 Tw5.84 Tw ex

1. Introduction: Number

- Mathematics is the science of patterns and relationships related to quantity. Listed below are some of the many relationships you may have come across
  - Every number is related to every other number in a number relationship. For example, 8 is 2 less than 10; made up of 4 and 4 (or 3 and 5); and is 10 times 0.8; is the square root of 64; and so on….
  - Number relationships are the foundation of strategies that help us remember number facts. For instance, knowing 4 + 4 = 8 allows one to quickly work out 4 + 5 (one more than 8); If one knows that 2 x 5 = 10, then 4 x 5 and 8 x 5 can easily be calculated (double 2 is 4 and so double 10 is 20; then double 4 is 8 and so double 20 is 40)
  - Each digit in a written numeral has a 'place' value which shows its relationship to '1'. For example, in 23.05 the value of the '2' is 20 ones, while the value of the '5' is only five hundredths of one. Understanding place value is critical to working with numbers.
- Mathematics is considered a universal language; however, words in English can often have more than one meaning which is why we sometimes find it difficult to translate from English to mathematical expressions.
- Arithmetic is a study of numbers and their manipulation.
- The most commonly used numbers in arithmetic are integers, which are positive and negative whole numbers including zero. For example, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6. Decimal fractions are not integers because they are 'parts of a whole', for instance, 0.6 is 6 tenths of a whole.
- The symbols we use between the numbers to indicate a task or relationships are the operands and the table below provides a list of common operators. You may recall the phrase 'order of operations.'

Example:

1. $12 \pm 6 = 6$ K N8.
2. $|F6| = 6$
3. 12.999 N13
4. 12 M7
5. Speed of light(speed of sound

1. Your Turn

Are the following statements true?

a) 32 M 4×8
b) $7 > 6$
c) 4 R4
d) $|5| = 5$
e) $37.1 + 22.02 = 59.3$

Integers

Whole numbers are integers; there are positive and negative integers. Positive integers are 1, 2, 3, 4, 5…
The negative integers are $-5, -4, -3, -2, -1$ (the dots before or after the sequence indicate that there are more numbers in this sequence that continue indefinitely)

Here are some more terms for you:

An equation implies that what is on either side of the '=' sign balances
The sum of two numbers implies two numbers are added together.

The sum of 4 and 8 is 12; $4 + 8 = 12$

The difference of two numbers implies that the second number is subtracted from the first number.

The difference between 9 and three is 6; $9 - 3 = 6$

The product of two numbers implies that two numbers are multiplied together.

The product of 3 and 4 is 12; $3 \times 4 = 12$

The quotient of two numbers implies that the first number is divided by the second.

The quotient of 20 and 4 is 5; $\frac{20}{4} = 5$ or $20 \div 4 = 5$

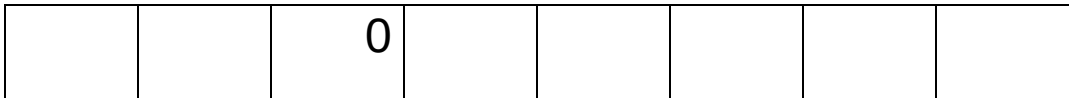
Rational Number: The term rational derives from the word ratio. Hence, a rational number can be a described by a ratio of integers or as a fraction. For example, $\frac{7}{8}$, J @75 are both rational numbers.

Irrational number: A number that cannot be written as a simple fraction or as a decimal fraction. If the number goes on forever without terminating, and without. - (s) 9 1

Here we are adding a positive number beginning at a negative number. Thus if we begin at zero we move in a negative direction to get to -2, and then in a positive direction of 6, and so we reach +4

(F2) + (+ 6) = 4

Your Turn: Represent F3 + 5 =



Subtraction:
To subtract a positive number we move that number of places to the left. For example, 5 F7, means we are subtracting a positive number, so we start at five and move 7 places to the left to get to F2. 5 F 7 = F2

Then to subtract a negative number we do the opposite and so we move to the right.

For example, (F2) F(F5) = 3 'negative 2 minus negative 5' which means (F2) + 5. Hence, subtracting a negative number is the same as adding a positive)

2. Rounding and Estimating

Rounding numbers is a method of decreasing the accuracy of a number to make calculations easier. Rounding is important when answers need to be given to a particular degree of accuracy. With the advent of calculators, we also need to be able to estimate a calculation to determine when the answer might be incorrect.

The Rules for Rounding:

1. Choose the last digit to keep.
2. If the digit to the right of the chosen digit is 5 or greater, increase the chosen digit by 1.
3. If the digit to the right of the chosen digit is less than 5, the chosen digit stays the same.
4. All digits to the right are now removed.

For example.3 ((R)]J -0.004 Tc 0.006 Tw 6.6[-3.5 (p)20.7 (o)1a8 (6)-6 (i)-3.3 (s)6.6- (t)-.6 (e)-7.7 (9d

3. Order of Operations

The order of operations matters when solving equations. Look at the example: $3 + 6 \times 2 = ?$

If I do the addition, then the multiplication, the answer would be: $9 \times 2 = 18$
If I do the multiplication, then the addition, the answer would be: $3 + 12 = 15$
There cannot be two answers to the same question. A rule is required to make sure everyone uses the same order.

There is a Calculation Priority Sequence to follow. Different countries, different states, even different teachers use a different mnemonic to help you remember the order of operations, but two common versions are BOMDAS and BIMDAS which stand for:

Brackets	{[( )]}
Other or Indices	T^6, $\sin T$, $\ln T$, $\sqrt{\ }$ A P ?
Multiplication or Division	× K N ÷
Addition or Subtraction	+ or -

The Rules:

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g. (× K N ÷) K (÷ K N ×)
3. If there are multiple brackets, work from the inside set of brackets outwards {[(ds)]}

Example Problems:

1. Solve: $5 + 7 \times 2 + 5^6 =$

Step 1: 5^6 has the highest priority so: $5 + 7 \times 2 + 25 =$

Step 2: 7×2 has the next priority so: $5 + 14 + 25 =$

Step 3: only addition left, thus left to right: $19 + 25 = 44$

$\therefore 5 + 7 \times 2 + 5^6 = 44$

2. Solve: $[(3 + 7) \times 6 \div 3] \times 7 =$

$[10 \times 6 \div 3] \times 7 =$

$[60 \div 3] \times 7 =$

$57 \times 7 = 399$

$\therefore [(3 + 7) \times 6 \div 3] \times 7 = 399$

3. Your Turn:

a) $4 \times (5 + 2) + 6 \div 12 \div 4 =$

b) $3 \times 7 + 6 \div 2 + 4 \div 2 + 7 =$

c) $2.4 \div 0.8 \times 5 + 8 \div 2 \times 6 =$

d) $(2.4 \div 0.8) \times 5 + 8 \times 6 \div 2 =$

Watch this short Khan Academy video for further explanation:
"Introduction to order of operations"
https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/algebra-foundations-order-of-operations/v/introduction-to-order-of-operations

4. Naming Fractions

- Fractions are representations of "even parts of a whole."
- A key concept is that division and fractions are linked. Even the division symbol is a fraction.

$$\frac{1}{2}$$

- A fraction is made up of two main parts: $\frac{7}{8}$

The denominator represents how many parts of the whole there are, and the numerator indicates how many of the parts are

5. Equivalent Fractions

Equivalence is a concept that is easy to understand when a fraction wall is used.

As you can see, each row has been split into different fractions: top row into 2 halves, bottom row 12 twelfths. An equivalent fraction splits the row at the same place. Therefore:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

The more pieces I split the row into (denominator), the more pieces I will need (numerator).

Mathematically, whatever I do to the numerator (multiply or divide), I must also do to the denominator and vice r3 (i)10.6 (v)-T /P <</(m) (e)-3 r

6. Converting Mixed Numbers to Improper Fractions

A mixed number is a way of expressing quantities greater than 1. A mixed number represents the number of wholes and remaining parts of a whole that you have, while an improper fraction represents how many parts you have. The diagram below illustrates the difference between a mixed number and an improper fraction, using a quantity of car oil as an example. On the left, we use a mixed number to represent 3 whole litres and 1 half litre. We write this mixed number as 3 ½. On the right, we use an improper fraction to represent 7 half litres. We write this improper fraction as $\frac{7}{2}$.

Is the same as

$$3\frac{5}{6} = \frac{7}{2}$$

You are more likely to encounter mixed numbers than improper fractions in everyday language. For example, you are more likely to say, 'my car requires 3 ½ litres of oil', rather than, 'my car requires $\frac{7}{2}$ litres of oil.'

It is much easier to multiply or divide fractions when they are in improper form. As such, mixed numbers are usually converted to improper fractions before they are used in calculations. To convert from a mixed number to an improper fraction, multiply the whole number by the denominator then add the numerator. This total then becomes the new numerator which is placed over the original denominator. For example:

Convert $3\frac{1}{2}$ into an improper fraction.

working: $3 \times 2 + 1 = 7$

Therefore, $3\frac{1}{2} = \frac{7}{2}$

Example problems:

1. $2\frac{6}{7} = \frac{20}{7}$ (2 × 3 + 2 = 8)

2. $2\frac{7}{8} = \frac{23}{8}$ (2 × 7 + 3 = 17)

6. Your Turn:

Convert these mixed numbers to improper fractions.

a) $4\frac{5}{6} =$ ___

c) $7\frac{7}{9} =$ ___

7. Converting Improper Fractions to Mixed Numbers

While improper fractions are good for calculations, they are rarely used in everyday situations. For example, people do not wear a size $\frac{67}{6}$ shoe; instead they wear a size $11\frac{5}{6}$ shoe.

$$= \frac{OA\ RA\ AD3HR\ A=O;}{6}$$

To convert to an improper fraction we need to work out how many whole numbers we have. Here we reverse the procedure from the previous section. We can see that 6 of the halves combine to form 3 wholes, with a half left over.

$$= 3\frac{1}{2}$$

So to work this symbolically as a mathematical calculation we simply divide the numerator by the denominator. Whatever the remainder is becomes the new numerator

Using a worked example of the diagram above. Convert $\frac{\cdot}{6}$

$7 \div 2 = 3\ \frac{5}{6}$ If I have three whole numbers, then I also have six halves and we have one half remaining. $\frac{\cdot}{6} = 3\ \frac{5}{6}$

8. Converting Decimals into fractions

Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, rather than fractions, into computers. But fractions can be more accurate. For example, $\frac{5}{7}$ is not 0.33 it is 0.33 6

The method used to convert decimals into fractions is based on the notion of place value. The place value of the last digit in the decimal determines the denominator: tenths, hundredths, thousandths, and so on…

Example problems:

1. 0.5 has 5 in the tenths column. Therefore, 0.5 is $\frac{9}{5\,4} = \frac{5}{6}$ (simplified to an equivalent fraction).
2. 0.375 has the 5 in the thousandth column. Therefore, 0.375 is $\frac{7\,;\,9}{5\,4\,4} = \frac{7}{<}$
3. = 7 ; 9

T /TT1 1 Tf -0.0045 Tc 0.006 Tw 11.016 96.0 Ì.984 (n) -0.7 (6 (D) 5 (7 (6.a (f)2th9 0.87 t tt tfh8-3.3 (n)-6 ( )10.9 (5-5-0.5(t)-(

d) 3.14 =

9. Converting Fractions into Decimals

From the image at right, we can see that we have three quarters of a whole cake. So to work this abstractly, we need to decide on a common denominator, which is 4.

Watch this short Khan Academy video for further explanation:
"Adding and subtraction fractions"
https://www.khanacademy.org/math/arithmetic/fractions/fractions-unlike-denom/v/adding-and-subtracting-fractions

11. Fraction Mult iplication and Division

Compared to addition and subtraction, multiplication and division of fractions is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have $\frac{5}{6}$ of a pie and I want to share it between 2 people. Each

Watch this short Khan Academy video for further explanation:
"Multiplying negative and positive fractions"
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-mult-div-frac/v/multiplying-negative-and-positive-fractions

Division of fractions seems odd, but it is a simple concept:

You may recall the expression 'invert and multiply' which means we flip the fraction; we switch the numerator and the denominator. Hence $\frac{5}{7} \div \frac{5}{6}$

12. Percentage

The concept of percentage is an extension of the material we have already covered about fractions. To allow comparisons between fractions we need to use the same denominator. As such, all percentages use 100 as the denominator. The word percent or “per cent” means per 100. Therefore, 27% is $\frac{27}{100}$.

To use percentage in a calculation, the simple mathematical procedure is modelled below:

For example, 25% of 40 is $\frac{25}{100} \times 40 = 10$

2. What do I need to score on the final exam to get a P, C, or a D? Can I still get a HD

	Calculation	Required score
P	For a Pass, I need to get at least 50% overall. I already have 36.67%, final exam needs to contribute 50 - 36.67 = 13.33	

13. Ratio

Understanding ratio is very closely related to fractions. With ratio, comparisons are made between equal sized parts/units. The ratio symbol is ‘ : ’ and it is used to separate the quantities being compared. For example, 1:3 means 1 unit to 3 units, and the units are the same size.

13. Your Turn (continued)

c) A tree shadow is 13m. A stick in the same vicinity is 30cm and casts a shadow of 50cm. How tall is the tree

d) A patient is prescribed 150mg of soluble aspirin. We only have 300mg tablets on hand. How many tablets should be given?

e) A solution contains fluoxetine 20mg/5mL. How many milligrams of fluoxetine are in 40mL of solution?

f) A stock has the strength of 5000units per mL. What volume must be drawn up into an injection to give 65

15. Powers

16. Power Operations

- Some basic rules :
  -

18. Root Operations

Some basic rules:

- $\sqrt{T} \times \sqrt{U} = \sqrt{T \times U}$ With numbers: $\sqrt{6} \times \sqrt{4} = \sqrt{6 \times 4} = \sqrt{24}$
- $\sqrt{T^2} = T$ With numbers: $\sqrt{5^2} = 5$

Simplifying roots requires finding factors which are square numbers (you have done this by finding some factors in the table on the previous page). Factors are all of the whole numbers that can be divided exactly into a number. For example, the factors of 12 are: 1, 2, 3, 4, 6, 12.

- Let's simplify $\sqrt{56}$; 56 has multiple factors and it is good practice to list them all:
  1 × 56
  2 × 28
  4 × 14 14 o itoe lify

20. Logarithms

With roots we tried to find the unknown base. For instance, $T^7 = 64$ is the same as $\sqrt[7]{64} = T$, (T is the base).

A logarithm is used to find an unknown power/exponent. For example, $4^{\square} = 64$ is the same as $\log_8 64 = T$

This example above is spoken as: The logarithm of 64 with base 4 is T. The base is written in subscript.

The general rule is: $0 = >^{\square} \Rightarrow \log_{\square} 0 = T$

- In mathematics the base can be any number, but only two types are commonly used:
  - $\log_{10} 0$ (>= O10A) is often abbreviated as simply Log, and
  - $\log_{e} 0$ (>= O eA) is often abbreviated as Ln or natural log
- $\log_{10} 0$ is easy to understand for $\log_{10} 1000 = \log 1000 = 3$ $(10^7 = 1000)$
  $\log 100 = 2$
- Numbers which are not 10, 100, 1000 and so on are not so easy. For instance, $\log 500 = 2.7$ It is more efficient to use the calculator for these types of expressions.

20. Your Turn:

Write an exponential equation for the following

a) $3 = \log_6 8$

b) $\frac{5}{6} = \log_{6\,9} 5$

c) $\log_{;} 49 = 2$

d) $\frac{7}{6} = \log_{5\,:} 64$

e) $\log_{<} \frac{5}{8} = F\frac{6}{7}$

Use a calculator to solve the following:

f) $\log 10000 =$

g) $\log 350 =$

h) $\ln 56 =$

i) $\ln 100 =$

Watch this short Khan Academy video for further explanation:
"Logarithms"
https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms

Some resources to help clarify using logs:

- Using logs in the real world

http://betterexplained.com/articles/using-logs-in-the-real-world/

- TED talk: Logarithms explained

http:// ed.ted.com/lessons/steve-kelly-logarithms-explained

21. Unit Conversions

Measurement is used every day to describe quantity. There are various types of measurements with various units of measure. For instance, we measure time, distance, speed, weight

Method 2

1. Write the conversion as a fraction.
2. Multiply out (include all units in the working).
3. Cancel any units that are both top and bottom.

Examples:

a) Convert 0.15g to milligrams

$$\frac{4\,5\,9Ú}{5} \times \frac{5\,4\,4à4Ú}{5Ú} = 150\ mg$$

b) Convert 5234 ml to litres

$$\frac{9\,6\,7à3Å}{5} \times \frac{5Å}{5\,4\,4à4Å} = 5.234\ L$$

c) Convert 21g/L to mg/ml

$$\frac{6\,5Ú}{5Å} \times \frac{5\,4\,4à4Ú}{5Ú} \times \frac{5Å}{5\,4\,4à4Å} = 21\ mg/mL$$

21. Your turn (continued)

Define the following SI unit prefixes, in words, as a number, and in exponential notation.

d) kilo

e) centi

10. Fraction Addition and Subtraction (page 13)

17. Roots (page 24)

18. Root Operations (page 25)

a. 9 b. $6\sqrt{2}$ c. $8\sqrt{3}$ d. $52\sqrt{2}$

19. Fraction Powers (page 26)

a. $\sqrt{81} = \pm 9$ b. $\frac{5}{\sqrt{5}} = \frac{5}{\pm\sqrt{5}} = \pm\frac{5}{\sqrt{5}}$

20. Logarithms (page 27)

a. $3 = \log_6 8 \;\wedge\; 2^7 = 8$

b. $\frac{5}{6} = \log_{6^6} 5 \;\wedge\; 25^{\frac{5}{6}} = 5$

$25^{\frac{5}{6}} \;\wedge\; \sqrt{25}$

c. $\log_{\frac{1}{7}} 49 = 2 \;\wedge\; 7^6 = 49$

d. $\frac{7}{6} = \log_{5^{\frac{1}{6}}} 64 \;\wedge\; 16^{\frac{7}{6}} = 64$

$16^{\frac{7}{6}} = 16^{7\times\frac{5}{6}}$

$\sqrt[6]{16^7} = \sqrt[4]{4096} = 64$

e. $\log_{\frac{1}{8}} = F\frac{6}{7} \;\wedge\; 8^{(?\frac{5}{7})} = \frac{5}{8}$

$\frac{1}{8^{\frac{6}{7}}} = \frac{1}{8^{6\times\frac{5}{7}}}$

$= \frac{1}{\sqrt{8^6}} = \frac{1}{\sqrt{64}}$

$= \frac{1}{4}$

f. 4

g. 2.54

h. 4.025

i. 4.605

21. Unit Conversions (page 29)

a. 0.285 GJ b. 0.00096 GJ or $9.6 \text{ GJ} \times 10^{?8}$ c. (0)Tj ()]TJ_0 1 Tf 0 Tc 0 Tw 8.04ET EMC /P <</MCID 48 >>

4.262 Tw 0.76.001 Tw 0.768 0 Td (262.44)-0.7(0)d (6)T1/C2_0 Tc 0 Tw 2.533 0 9d (0)T

21. Unit Conversions (continued) (page 32)

d. kilo one thousand, 1 000, 10^{3}

e. centi one hundredth, 0.01, 10^{-2}

f. mega one million, 1 000 000, 10^{6}

g. deci one tenth, 0.1, 10^{-1}

h. 1.00×10^{3} L. $\frac{\text{1 L}}{\text{1 000 mL}} \times \frac{\text{1 000 mL}}{\text{1 L}} \times \frac{\text{1 000 mL}}{\text{1 L}} = 1000\ 000\text{mL}$ or 1.00×10^{6} mL.

i. 15 inches: if 2.54cm = 1 inch, then $\frac{\text{15 inches}}{\text{1 inch}} \times \frac{\text{2.54 cm}}{\text{1 inch}} = 15 \times 2.54\text{ cm}$

j. 1.14×10^{5} mL: $\frac{\text{1 L}}{\text{1 000 mL}} \times \frac{\text{1 000 mL}}{\text{1 L}} \times \frac{\text{1 000 mL}}{\text{1 L}} = 114000$ mL

k. 350 litres: $\frac{\text{1 000 mL}}{\text{1 L}} \times \frac{\text{1 L}}{\text{1 000 mL}} = 350$

23. Glossary

Addition